(In)Stability in Networked Markets: Supplemental Material

For Online Publication

Zeky Murra-Anton* ⓡ Andrew Ferdowsian†

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Abstract

This online appendix contains four supplementary items. Online Appendix A compares our framework with the closest related solution concepts in terms of their primitives, deviations, post-deviation responses, and solution objects. Online Appendix B constructs an economy that features no market gate or concentrated components, yet is structurally unstable. Online Appendix C constructs an economy with core preservation under a punishment rule that violates disruption neutrality. Online Appendix D provides the calculations supporting the centralization-limits remark in the main text.

*Market Development, ISO New England. zmurraanton@iso-ne.com. This paper was not endorsed or funded by ISO New England.

†Department of Economics, University of Notre Dame: pingandrew@gmail.com.

A. Comparison with Adjacent Solution Concepts

This section compares the network-disruption framework with the closest approaches to cooperative externalities. The comparison separates four objects: the primitive environment, the deviation technology, the post-deviation response, and the resulting solution concept.

Table 1: Adjacent solution concepts

Framework	**Function**	**Comparison**
Partition-function games (Thrall and Lucas, 1963)	A partition function assigns a worth to coalition S as a function of the coalition structure containing it. Thus the same coalition may have different worths depending on how the remaining players organize. A solution concept for partition-function games must therefore specify how a deviation changes the residual coalition structure or how outsiders respond.	**Commonality.** Link withholding creates the same basic externality: a coalition's action changes the feasible opportunities of other coalitions. **Difference.** The primitive consequence of a network-disruption is a new active network $\hat{L}$, not a partition of the player set. Conditional on $\hat{L}$, all players remain in a fixed-network settlement game. A settlement rule selects a core payoff of that game, generating a characteristic function. Our framework focuses on this structured class of feasibility externalities for tractability; the framework is not a reduction of arbitrary partition-function games.

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Framework	**Function**	**Comparison**
Alpha- and beta-effectiveness (Aumann and Peleg, 1960)	The primitive is a fixed strategic game. Under α-effectiveness, upon deviating coalition S requires a strategy that attains the target payoff against every strategy of the coalition's complement. β-effectiveness reverses the qualifier order: For every strategy of the complement, coalition S requires some strategy that attains the target payoff. The corresponding core notions exclude allocations dominated by coalitions that are effective in the relevant sense.	**Commonality.** Both frameworks discipline a coalition's deviation by specifying what it can obtain in light of a response by nonmembers. The punishment rule is similarly conservative because it assigns the deviators the lowest aggregate payoff available in the induced settlement core. **Difference.** α and β effectiveness quantify over strategies in a fixed strategic game. Our framework involves a coalition first changing the feasible network, then facing a response which is a payoff selected from the core of a new cooperative game. In particular, the α and β core permit punishment via incredible threats, whereas our framework requires punishments to be drawn from the core of the new network. There is no general equivalence between network stability and either core notion.

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Framework	**Function**	**Comparison**
Gamma-characteristic function (Chander and Tulkens, 1997)	In a production game with externalities, a deviating coalition acts jointly while outsiders individually take best-response actions. Those responses produce a scalar worth for each coalition, to which an ordinary core analysis is then applied.	**Commonality.** Both approaches recover a characteristic function by specifying a disciplined post-deviation response in an environment with externalities. **Difference.** The gamma construction imposes a particular Nash-like behavior on outsiders. The settlement rule here may select any payoff in the core of the induced fixed-network market and may condition on both the withholding coalition and the induced network. The restriction is self-enforcement of the entire induced settlement, as opposed to a prescribed individual equilibrium response by outsiders.

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Framework	**Function**	**Comparison**
Recursive core (Huang and Sjöström, 2006; Kóczy, 2007)	After coalition S deviates, the residual players face a smaller partition-function game. Their response is determined recursively by the solution of that residual game.	**Commonality.** Both frameworks reject arbitrary post-deviation reactions and require the continuation to be internally consistent. **Difference.** Our framework does not recursively solve a residual coalition-formation problem. Withholding changes the active network, after which the full set of players settles in the core of the induced fixed-network game $\Gamma(\mathcal{E}, \hat{L})$. The settlement rule then prices the deviators' payoff in that market. This nonrecursive reduction yields an ordinary TU game $\Psi(\mathcal{E}, \sigma)$ and permits the Bondareva–Shapley characterization.

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Framework	Function	Comparison
Coalitional credibility and farsighted stability (Bernheim et al., 1987; Chwe, 1994; Konishi and Ray, 2003; Ray and Vohra, 1997, 2015)	Coalition-proof Nash equilibrium requires proposed strategic deviations to be self-enforcing. Equilibrium binding agreements model the behavior of the complement explicitly after a coalition changes the coalition structure. Farsighted and dynamic models evaluate coalitional moves through subsequent reactions or continuation values, and coalitional-sovereignty approaches restrict a deviating coalition's ability to dictate outsiders' payoffs.	**Commonality.** Again, both approaches share the concern that a deviation cannot be evaluated using arbitrary or noncredible continuation behavior. **Difference.** Network stability does not refine a strategy profile, recursively form coalitions, or evaluate chains of future moves. It treats the observable withholding event $(S, \hat{L})$ as the within-window deviation and evaluates the self-enforcing settlement on the induced active network.

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Framework	**Function**	**Comparison**
Perfect coalitional equilibrium (Ali and Liu, 2026)	Perfect coalitional equilibrium is a repeated-game solution concept. A history-dependent plan specifies continuation behavior after every history, and no coalition can profitably block at any history once the current gain and the induced continuation payoff are taken into account. Future rewards and punishments are themselves required to be credible.	**Commonality.** PCE is the closest dynamic counterpart to the paper's concern with credible responses to coalitional deviations. Both rule out punishments that are not self-enforcing. **Difference.** A settlement rule assigns a payoff within a single clearing-and-settlement window after the active network is realized; the model contains no discounting, history dependence, or future punishment. The two models operate on fundamentally different objects; our model uses a one-shot interaction, while PCE involves a repeated game.

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Framework	Function	Comparison
Biform games (Brandenburger and Stuart, 2007)	A biform game combines a noncooperative first stage with a cooperative second stage. An individual strategic-action profile determines the cooperative game that follows. A value-capture specification or a range derived from the second-stage core then determines first-stage payoffs, and the first-stage game is analyzed using a noncooperative equilibrium concept.	**Commonality.** In both frameworks, actions alter the cooperative environment that is subsequently relevant for payoff determination. **Difference.** The noncooperative nature of the initial stage in biform games radically differs from our cooperative nature of link withholding in the first stage. For instance, were link withholding conducted in a noncooperative manner, third-party right's holders would have no incentive to ever withhold links.

The distinctive structure of our framework is, therefore, the combination of three elements. First, a deviation changes feasibility by inducing an active network. Second, the subsequent post-deviation response must be a core allocation of the induced fixed-network settlement game. Third, once a settlement rule is fixed, every coalition's rematching and withholding opportunities are summarized by a scalar worth $U_{\mathcal{E},\sigma}$. The resulting network-stability problem is exactly the core problem for the network disruption game $\Psi(\mathcal{E},\sigma)$. None of the comparison frameworks combines these three steps in this form.

B. Dispersed internal rights

Consider the punishment rule σ^P. Let $G = \{g_1, g_2, g_3\}$, $I = \{i_1, i_2\}$, and $P = \emptyset$. Let the cross-links be

$$E = \{(g_1, i_1), (g_1, i_2), (g_2, i_1), (g_3, i_2)\},$$

with access rights

$$\tau(g_1, i_1) = i_1, \quad \tau(g_1, i_2) = i_2, \quad \tau(g_2, i_1) = g_2, \quad \tau(g_3, i_2) = g_3.$$

Thus every cross-link is controlled by one of its endpoints, and no agent controls more than one cross-link. Let

$$v_{(g_1,i_1)} = v_{(g_1,i_2)} = v_{(g_2,i_1)} = v_{(g_3,i_2)} = 1, \quad v_l = 0 \text{ for all } l \in E^0.$$

Under the full network, at most two positive-value links can be used simultaneously, so

$$U_{\mathcal{E},\sigma^P}(N_{\mathcal{E}}) = V_{\mathcal{E},L}(N_{\mathcal{E}}) = 2.$$

Let $S^* = \{g_1, g_2, g_3\}$. Since S^* contains no firm, $V_{\mathcal{E},L}(S^*) = 0$. But S^* can withhold (g_2, i_1) and (g_3, i_2), inducing an active network $\hat{L}$ in which the only positive-value cross-links are (g_1, i_1) and (g_1, i_2). These two links cannot be used simultaneously, so $V_{\mathcal{E},\hat{L}}(N_{\mathcal{E}}) = 1$.

For any $u \in Core(\Gamma(\mathcal{E}, \hat{L}))$, the core constraints for $\{g_1, i_1\}$ and $\{g_1, i_2\}$ imply

$$u_{g_1} + u_{i_1} \geq 1 \quad \text{and} \quad u_{g_1} + u_{i_2} \geq 1.$$

Core efficiency gives $\sum_{n \in N_{\mathcal{E}}} u_n = 1$, and the singleton core constraints imply $u_n \geq 0$ for every player. Hence $u_{g_1} = 1$ in the unique core allocation of $\Gamma(\mathcal{E}, \hat{L})$. Therefore,

$$U_{\mathcal{E},\sigma^P}(S^*) \geq B_{\mathcal{E},\sigma^P}(S^*) \geq 1.$$

For each cross-link $(g_k, i_l) \in E$, let $S_{kl} = \{g_k, i_l\}$. Each such coalition can realize one positive-value link under the full network. Therefore,

$$U_{\mathcal{E},\sigma^P}(S_{11}) \geq 1, \quad U_{\mathcal{E},\sigma^P}(S_{12}) \geq 1, \quad U_{\mathcal{E},\sigma^P}(S_{21}) \geq 1, \quad U_{\mathcal{E},\sigma^P}(S_{32}) \geq 1.$$

Consider the balanced weights

$$\delta(S^*) = \delta(S_{11}) = \delta(S_{12}) = \frac{1}{3}, \quad \delta(S_{21}) = \delta(S_{32}) = \frac{2}{3}, \quad \delta(S) = 0 \text{ otherwise.}$$

Then

$$\sum_{\emptyset \neq S \subsetneq N_{\mathcal{E}}} \delta(S) U_{\mathcal{E},\sigma^P}(S) \geq \frac{7}{3} > 2 = U_{\mathcal{E},\sigma^P}(N_{\mathcal{E}}).$$

Thus $\Psi(\mathcal{E}, \sigma^P)$ is not balanced. By Corollary 1, $NS(\mathcal{E}, \sigma) = \emptyset$ for every settlement rule σ for $\mathcal{E}$. Moreover, the violation of balancedness is strict. By the continuity argument in the proof of Theorem 2, it persists in a nonempty open neighborhood of the displayed surplus vector. Hence, this network structure is structurally unstable.

C. Full-Core Preservation without Disruption Neutrality

Theorem 3 establishes that disruption neutrality is sufficient for the preservation of the entire fixed-network core; however, disruption neutrality is not necessary. Consider an economy with $G = \{g_1, g_2, g_3\}$, $I = \{i\}$, and $P = \emptyset$. The cross-links are

$$\ell_1 = (g_1, i), \qquad \ell_2 = (g_2, i), \qquad \ell_3 = (g_3, i),$$

all controlled by firm i, with

$$v_{\ell_1} = 10, \qquad v_{\ell_2} = 9, \qquad v_{\ell_3} = 1,$$

and all self-link values equal to zero.

We first characterize the core of the full-network settlement game. Core efficiency requires

$$u_{g_1} + u_{g_2} + u_{g_3} + u_i = 10.$$

The core constraint of coalition $\{g_1, i\}$ requires

$$u_{g_1} + u_i \geq 10.$$

Together with the nonnegativity implied by the singleton core constraints, these conditions give

$$u_{g_2} = u_{g_3} = 0 \quad \text{and} \quad u_{g_1} + u_i = 10.$$

The core constraint of coalition $\{g_2, i\}$ further requires $u_i \geq 9$. Conversely, these conditions satisfy every full-network core constraint. Hence, with coordinates ordered as (g_1, g_2, g_3, i),

$$Core(\Gamma(\mathcal{E}, L)) = \{u(x) = (x, 0, 0, 10 - x) : x \in [0, 1]\}.$$

We next show the punishment rule is not disruption neutral. Consider coalition $S^0 = \{i\}$, whose full-network value is $V_{\mathcal{E},L}(S^0) = 0$. Coalition S^0 can withhold ℓ_3, inducing the active network

$$\hat{L} = L \setminus \{\ell_3\}.$$

Let $y \in Core(\Gamma(\mathcal{E}, \hat{L}))$. Core efficiency gives

$$y_{g_1} + y_{g_2} + y_{g_3} + y_i = 10.$$

The core constraint of $\{g_1, i\}$ implies

$$y_{g_1} + y_i \geq 10,$$

so nonnegativity gives $y_{g_2} = y_{g_3} = 0$ and $y_{g_1} + y_i = 10$. The core constraint of $\{g_2, i\}$ then implies

$$y_i \geq 9.$$

This lower bound is attained at

$$(y_{g_1}, y_{g_2}, y_{g_3}, y_i) = (1, 0, 0, 9).$$

Therefore,

$$\min_{y \in Core(\Gamma(\mathcal{E}, \hat{L}))} y_i = 9,$$

and hence

$$B_{\mathcal{E},\sigma^P}(\{i\}) \geq 9 > 0 = V_{\mathcal{E},L}(\{i\}).$$

Thus σ^P is not disruption neutral.

Nevertheless, the punishment rule preserves the entire full-network core. To see it, fix any $u(x) \in Core(\Gamma(\mathcal{E}, L))$ and any coalition $S \in \mathcal{S}^R_{\mathcal{E}}$. Since firm i controls every cross-link, $i \in S$.

Suppose first that $g_1 \in S$. Then

$$\sum_{n \in S} u_n(x) = u_{g_1}(x) + u_i(x) = 10.$$

For every active network $\hat{L}$, link withholding cannot increase the grand coalition's surplus above 10. Moreover, all induced-core payoffs are nonnegative. Therefore,

$$\sum_{n \in S} \sigma_n^P(\hat{L}, S) \leq 10$$

for every $\hat{L} \in \mathcal{L}_{\mathcal{E}}(S)$, and hence

$$B_{\mathcal{E},\sigma^P}(S) \leq 10 = \sum_{n \in S} u_n(x).$$

Now suppose that $g_1 \notin S$. Then

$$\sum_{n \in S} u_n(x) = u_i(x) = 10 - x \geq 9.$$

Now fix any $\hat{L} \in \mathcal{L}_{\mathcal{E}}(S)$. If $\ell_1 \notin \hat{L}$, then the grand coalition's induced-network value is at most 9. Nonnegativity and core efficiency therefore imply

$$\sum_{n \in S} \sigma_n^P(\hat{L}, S) \leq 9.$$

Suppose instead that $\ell_1 \in \hat{L}$. Let

$$r(\hat{L}) = \max\bigl(\{v_\ell : \ell \in \hat{L} \cap \{\ell_2, \ell_3\}\} \cup \{0\}\bigr),$$

so $r(\hat{L}) \in \{0, 1, 9\}$. Define $y^{\hat{L}}$ by

$$y_{g_1}^{\hat{L}} = 10 - r(\hat{L}), \qquad y_i^{\hat{L}} = r(\hat{L}), \qquad y_{g_2}^{\hat{L}} = y_{g_3}^{\hat{L}} = 0.$$

This vector is efficient, every coalition containing g_1 and i receives 10, every coalition not containing i has value zero, and every coalition containing i while lacking g_1 can generate at most $r(\hat{L})$. Therefore, $y^{\hat{L}} \in Core(\Gamma(\mathcal{E}, \hat{L}))$. Since $g_1 \notin S$,

$$\sum_{n \in S} y_n^{\hat{L}} = r(\hat{L}) \leq 9.$$

The punishment rule minimizes the aggregate payoff of S over the induced core, so

$$\sum_{n \in S} \sigma_n^P(\hat{L}, S) \leq 9.$$

It follows that

$$B_{\mathcal{E},\sigma^P}(S) \leq 9 \leq \sum_{n \in S} u_n(x).$$

Thus every payoff in $Core(\Gamma(\mathcal{E}, L))$ satisfies every inter-network stability constraint under σ^P. Since network stability always requires membership in the full-network core,

$$NS(\mathcal{E}, \sigma^P) = Core(\Gamma(\mathcal{E}, L)).$$

In particular,

$$B_{\mathcal{E},\sigma^P}(\{i\}) = 9 > 0 = V_{\mathcal{E},L}(\{i\}),$$

even though the entire full-network core is preserved. The reason is that coalition $\{i\}$ receives at least 9 at every full-network core payoff:

$$\min_{u \in Core(\Gamma(\mathcal{E},L))} u_i = 9.$$

The disruption rent, therefore, exhausts slack already present in the coalition's full-network core constraint without eliminating any full-network core payoff.

D. Limits of Centralization

Centralization need not imply disruption neutrality or preservation of the entire fixed-network core. To see this, let $G = \{g_1, g_2\}$, $I = \{i_1, i_2\}$, and $P = \{t\}$. Let the cross-links be

$$\ell_1 = (g_1, i_1), \quad \ell_2 = (g_1, i_2), \quad \ell_3 = (g_2, i_2),$$

all controlled by t, with

$$(v_{\ell_1}, v_{\ell_2}, v_{\ell_3}) = (9, 9, 1),$$

and let all self-links have zero value.

Under the full network, the efficient matching uses ℓ_1 and ℓ_3, so

$$V_{\mathcal{E},L}(N_{\mathcal{E}}) = 10.$$

Consider the payoff vector u defined by

$$u_t = 8, \qquad u_{i_1} = u_{i_2} = 1, \qquad u_{g_1} = u_{g_2} = 0.$$

This payoff vector is efficient. We verify that it is also core-stable. Any coalition not containing t controls no cross-link and has value zero, so its core constraint follows from the nonnegativity of u. Now consider a coalition containing t. It is enough to check the possible sets of positive-value cross-links used by a feasible matching; omitted self-links have zero value. Under u,

cross-links used M	$v(M)$	$\sum_{n \in S(M)} u_n$
$\{\ell_1\}$	9	9
$\{\ell_2\}$	9	9
$\{\ell_3\}$	1	9
$\{\ell_1, \ell_3\}$	10	10,

where $S(M)$ consists of t and the trading agents incident to the links in M. For any coalition S and any matching feasible for S, we have $S(M) \subseteq S$. Since u is nonnegative,

$$\sum_{n \in S} u_n \geq \sum_{n \in S(M)} u_n \geq v(M).$$

Maximizing over the matchings feasible for S gives

$$\sum_{n \in S} u_n \geq V_{\mathcal{E},L}(S).$$

Hence

$$u \in Core(\Gamma(\mathcal{E}, L)).$$

Now let $S^* = \{g_1, t\}$ withhold ℓ_3, inducing the active network

$$\hat{L} = L \setminus \{\ell_3\}.$$

In the induced network, the efficient matching uses either ℓ_1 or ℓ_2, so

$$V_{\mathcal{E},\hat{L}}(N_{\mathcal{E}}) = 9.$$

Let $y \in Core(\Gamma(\mathcal{E}, \hat{L}))$. The coalitions

$$S(\{\ell_1\}) = \{g_1, i_1, t\} \quad \text{and} \quad S(\{\ell_2\}) = \{g_1, i_2, t\}$$

can realize ℓ_1 and ℓ_2, respectively, each worth 9. Thus

$$y_{g_1} + y_{i_1} + y_t \geq 9 \quad \text{and} \quad y_{g_1} + y_{i_2} + y_t \geq 9.$$

Core efficiency gives

$$\sum_{n \in N_{\mathcal{E}}} y_n = 9,$$

while the singleton core constraints imply $y_n \geq 0$ for every player. The first inequality therefore implies

$$y_{g_2} = y_{i_2} = 0,$$

and the second implies

$$y_{g_2} = y_{i_1} = 0.$$

Hence

$$y_{i_1} = y_{i_2} = y_{g_2} = 0,$$

and core efficiency gives

$$y_{g_1} + y_t = 9 \quad \text{for every } y \in Core(\Gamma(\mathcal{E}, \hat{L})).$$

Because S^* contains no firm,

$$V_{\mathcal{E},L}(S^*) = 0.$$

Every settlement rule selects a payoff in $Core(\Gamma(\mathcal{E}, \hat{L}))$, so, for every settlement rule

σ,

$$B_{\mathcal{E},\sigma}(S^*) \geq \sum_{n \in S^*} \sigma_n(\hat{L}, S^*) = 9 > 0 = V_{\mathcal{E},L}(S^*).$$

Therefore no settlement rule is disruption neutral. In particular,

$$\Delta_{\mathcal{E},\sigma^P}(S^*) > 0.$$

Moreover, the full-network core payoff u gives coalition S^* only

$$u_{g_1} + u_t = 8.$$

Since every settlement rule gives S^* a payoff of 9 after it induces $\hat{L}$,

$$u_{g_1} + u_t = 8 < 9 = \sum_{n \in S^*} \sigma_n(\hat{L}, S^*)$$

for every settlement rule σ. Hence

$$u \notin NS(\mathcal{E}, \sigma) \quad \text{for every settlement rule } \sigma.$$

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