

Both water drag and air drag on a kayak are roughly proportional to the square of speed. So water drag (D_w) and air drag (D_a) are

$$D_w = B \cdot RWS^2 \quad D_a = b \cdot RAS^2$$

where RWS is the kayak's relative water speed (speed of kayak relative to the moving water) and RAS is the relative air speed (air speed relative to moving kayak). Also, B and b are the proportionality constants.

At a speed of 2 knots the water drag on a typical sea kayak is 1 pound (see <http://keelhauler.org/khcc/seakayak.htm>). The air drag (for 4 sq. feet area) is .007 at a wind speed of 1 knot. So, using these values we can compute B and b as follows:

$$1 = B \cdot 2^2 \quad .007 = b \cdot 1^2$$

$$B = .25 \quad b = .007 \quad \text{Note that } B \gg b$$

Case 1: DownCurrent/UpWind

Given a current speed C, a wind speed A and the kayak's speed-over-ground, SOG, the kayak's relative water speed and the relative wind speed are:

$$RWS = SOG - C \quad RAS = SOG + A$$

So, as a function of SOG, the total drag on the kayak for case 1 is

$$TD(SOG) = B \cdot (RWS)^2 + b \cdot (RAS)^2 = B \cdot (SOG - C)^2 + b \cdot (SOG + A)^2$$

Using the values calculated for b and B we have for the case 1 total drag

$$TD(SOG) = .25 \cdot (SOG - C)^2 + .007 \cdot (SOG + A)^2 \quad \text{eq. 1}$$

Case 2: UpCurrent/DownWind

With the same notation as above we have

$$RWS = SOG + C \quad RAS = SOG - A$$

So, as a function of SOG, the total drag on the kayak for case 2 is

$$TD(SOG) = B \cdot (RWS)^2 + b \cdot (RAS)^2 = B \cdot (SOG + C)^2 + b \cdot (SOG - A)^2$$

$$TD(SOG) = .25 \cdot (SOG + C)^2 + .007 \cdot (SOG - A)^2 \quad \text{eq. 2}$$

Subtracting eq.1 from eq.2 we get

$$\left[.25 \cdot (\text{SOG} + C)^2 + .007 \cdot (\text{SOG} - A)^2 \right] - \left[.25 \cdot (\text{SOG} - C)^2 + .007 \cdot (\text{SOG} + A)^2 \right]$$

which simplifies to

$$4 \cdot \text{SOG} \cdot (.25 \cdot C - .007 \cdot A)$$

And plugging in 2 for C and 20 for A we get

$$1.44 \cdot \text{SOG} \quad \text{eq. 3}$$

Since eq. 3 is always non-negative (because SOG is non-negative) the drag for case 2 is greater than for case 1 and increases linearly with increasing SOG. In fact, as we get above hull speed, the exponent in the water drag equation is larger than 2.0 (the air drag exponent is always 2.0) so the result in that situation is that the drag for case 2 is even more larger than it would be for case 1 below hull speed.